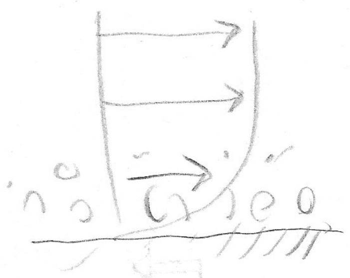
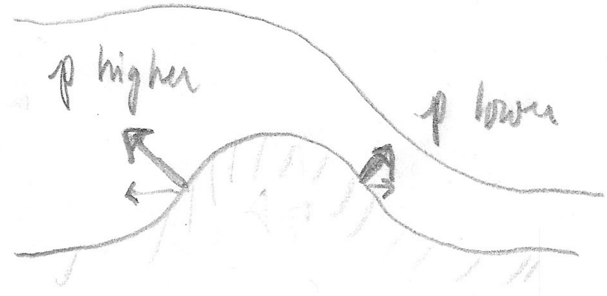
(9)

$\boxed{\text{Form Drag}}$

7/21/2019

1





Boundary Layer
Friction pushes
on flow

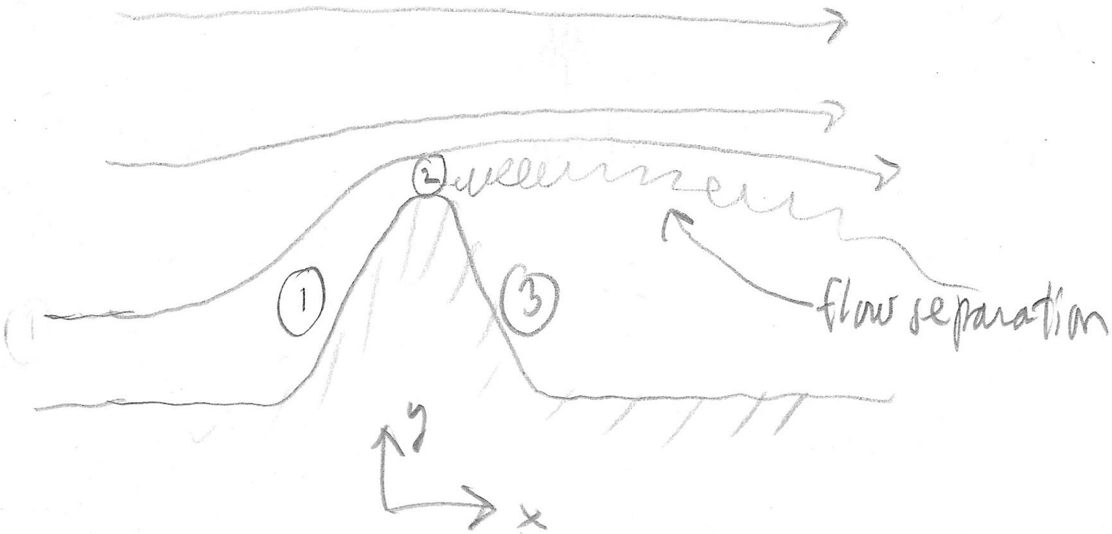
- Tangential Stress

As does rough topography
Through pressure x slope

- Normal Stress

Form drag:
- often $\gg C_d u^2$
- arises from flow separation, hydraulics, and internal wave generation
- almost always associated w/ dissipation

Consider SW Flow Past a Headland



Bernoulli $\frac{1}{2}u^2 + g\eta$ = const. on streamline

1 u ~ slow $\Rightarrow$ η ~ high

2 u ~ fast $\Rightarrow$ η ~ low

3 u ~ 0 But we can't use Bernoulli (line integral crosses vorticity of flow sep.)

Instead assume η same on either side of flow separation.

Force of Headland on Fluid

Force of fluid on Headland

¿Where is momentum lost in fluid?

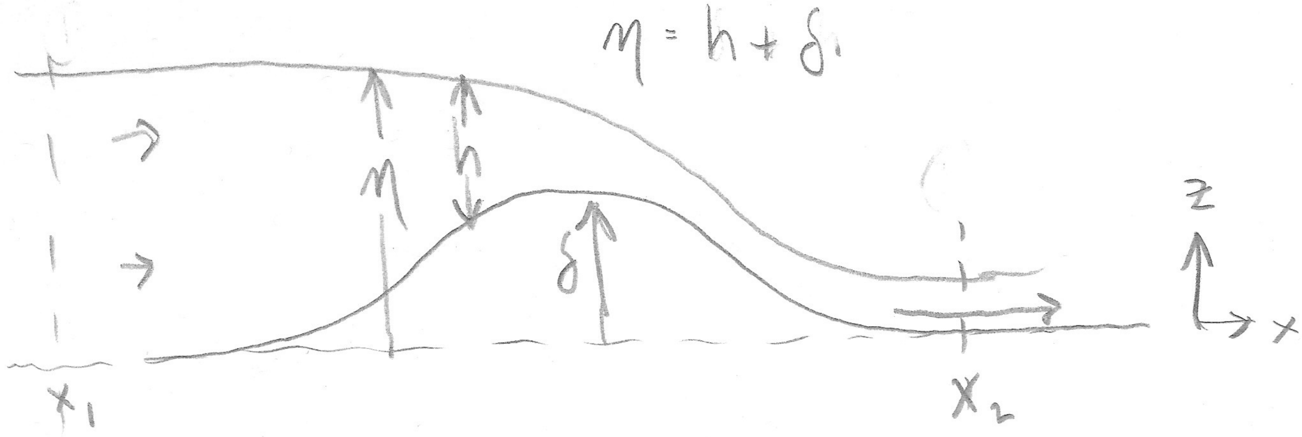
in the "headland eddy"

Where is energy lost?

Formally, form drag is a term in

$$\int \boxed{x\ \text{mom}}\, dV$$

Simple Case: Hydraulic SW Flow over Bump



$\boxed{x\ \text{mom}}$ $\quad u u_x + g \eta_x = 0$

$\boxed{\text{mass}}$ $\quad (hu)_x = 0$

Take vertical integral:

$$h u u_x + g h \eta_x = 0$$

$$\Rightarrow (huu)_x - \underbrace{(\cancel{h}u)_x}_{0} u + g h h_x + g h \delta_x = 0$$

$$\Rightarrow \left(h u^2 + \tfrac{1}{2} g h^2\right)_x + g h \delta_x = 0$$

Take x - integral :

$$hu^2 \Big|_{x_1}^{x_2} + \frac{1}{2} g h^2 \Big|_{x_1}^{x_2} + \int_{x_1}^{x_2} g h \, \delta_x \, dx = 0$$

Net mom. flux through ends

Net pressure force on ends

Form Drag ~ bottom pressure × bottom slope

☆ • What are the units?

A: $\frac{\text{Vol}}{\text{unit } y} \; \frac{\text{Force}}{\text{unit mass}} \sim \frac{L^3}{L} \sim \frac{ML^2}{T^2} \frac{1}{M} \sim \frac{L^3}{T^2}$

• Which direction are these three forces pushing?

Form Drag: Formal Derivation from full momentum integral

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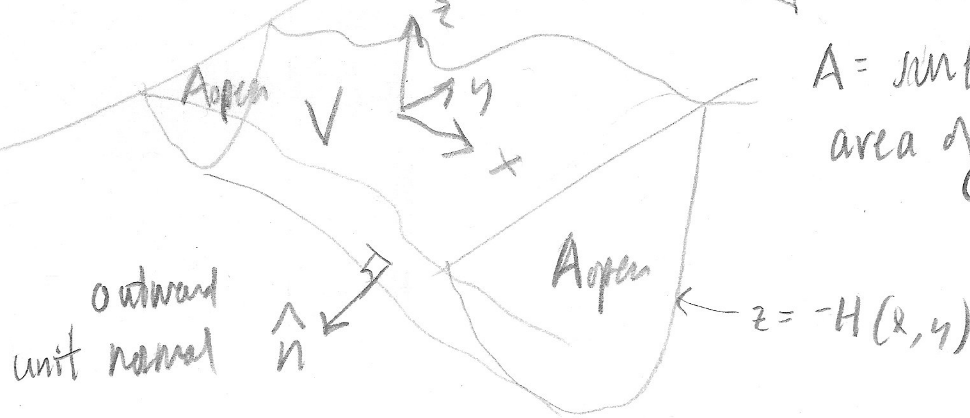


A = surface area of V

$\boxed{\underline{x}\text{ mom}}$: $$\frac{D\underline{u}}{Dt} + f\hat{k}\times\underline{u} = -\frac{1}{\rho_0}\nabla p - \frac{\hat{k}g\rho}{\rho_0} + \nabla\cdot(A\nabla\underline{u})$$

First step:

$$\int_V \boxed{\underline{x}\text{ mom}}\, dV : \quad \overset{(1)}{\frac{d}{dt}\int_V \underline{u}\, dV} + \overset{(2)}{\int_{A_{open}} \underline{u}\, u_n\, dA} = \overset{(3)}{-\frac{1}{\rho_0}\int_A p\,\hat{n}\, dA}$$

$$- \int_V \frac{\hat{k}g\rho}{\rho_0}\, dV + \overset{(4)}{\int_A (A\nabla\underline{u})\cdot\hat{n}\, dA}$$

We have used 3 relations worth memorizing:

(i) $$\int_V \frac{D\overset{\text{scalar field}}{\varphi}}{Dt}\, dV = \frac{d}{dt}\int_V \varphi \, dV + \int_{A_{open}} \varphi \, \overbrace{\underline{u}\cdot\hat{n}}^{\equiv u_n} \, dA$$

$$\left.\begin{aligned}
&\text{(ii)} \quad \int_V \nabla\varphi \, dV = \int_A \varphi \, \hat{n} \, dA \\
&\text{(iii)} \quad \int_V \nabla\cdot\overset{\text{vector field}}{\underline{a}} \, dV = \int_A \underline{a}\cdot\hat{n} \, dA
\end{aligned}\right\} \text{Gauss' theorem (Gauss' Divergence Theorem)}$$

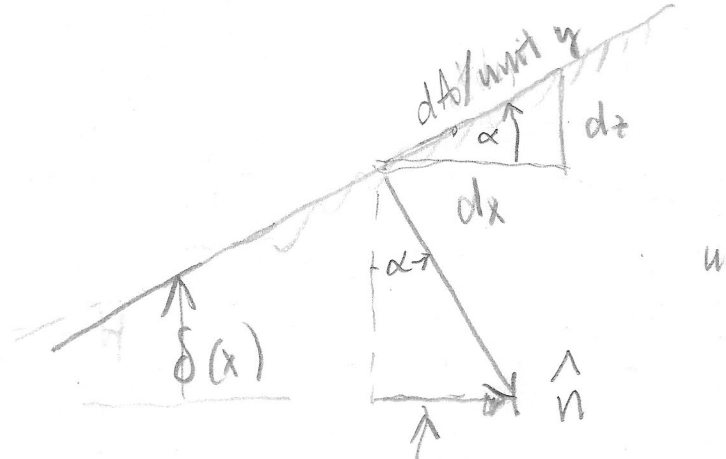
Second step: take $\hat{i} \cdot \int_V \boxed{\underline{x}\ \text{mom}}\, dV$

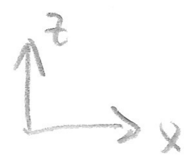
$$\Rightarrow \overset{(1)}{\frac{d}{dt}\int u\, dV} + \overset{(2)}{\int_{A_{open}} u u_n \, dA} = \overset{(3)}{-\frac{1}{\rho_0}\int_A p \, \hat{i}\cdot\hat{n} \, dA}$$

$$+ \overset{(4)}{\int_A (A \nabla u)\cdot \hat{n} \, dA}$$

The part of (3) on the seafloor is Form Drag.

Note:





$\frac{dA}{\text{unit } y} = \frac{dx}{\cos\alpha}$

$\hat{i}\cdot\hat{n} = \sin\alpha$

$\Rightarrow \hat{i}\cdot\hat{n}\ dA/\text{unit } y$

$= \frac{dx}{\text{unit } y}(\tan\alpha)$

$= \frac{\partial \delta}{\partial x} \frac{dx}{\text{unit } y}$

like the pressure work term in flow over bump

☆

- Describe 1 - 4 in words
- what happened to gravity term?